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ELECTROMAGNETIC DESCRIPTION OF THE TWO STREAM INSTABILITY IN A RELATIVISTIC ELECTRON BEAM

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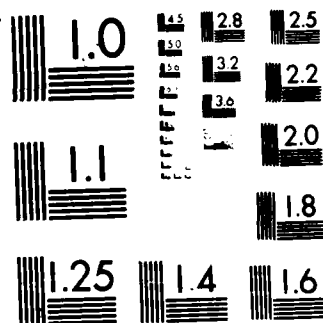## NAVAL SURFACE WEAPONS CENTER
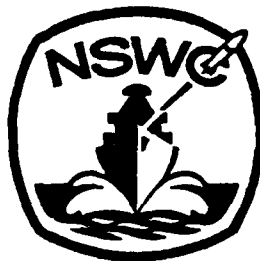
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ELECTROMAGNETIC DESCRIPTION OF THE TWO STREAM INSTABILITY IN A RELATIVISTIC ELECTRON BEAM

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The influence of electromagnetic effects on the two stream instability in a relativistic electron beam propagating through a collisionless plasma channel is examined, and closed algebraic dispersion relation for the complex eigenfrequency ω is obtained. It is shown that electromagnetic effects can have a strong stabilizing influence on the two stream instability in a relativistic electron beam, drastically enhancing the critical beam current for instability.

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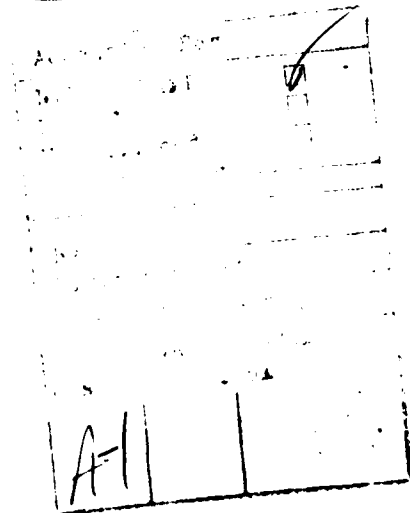
One of the most basic instabilities that characterize a relativistic electron beam propagating through a collisionless plasma is the two stream instability,[1-3] which resulted from the relative drift motion between the beam electrons and the background plasma particles. Although the two stream instability is relatively familiar in the plasma physics community, most of the previous studies[1,2] on this instability have been limited to one dimensional calculations. In recent literature,[3] Bogdankevich and Rukhadze investigated the two stream instability of a relativistic electron beam, including the finite radial geometry effects on stability behavior. Thus, they were able to determine the limiting beam current due to the two stream instability. However, their calculation has been based on the electrostatic approximation. Although this is a reasonable approximation for a mildly relativistic electron beam, a significant modification to the stability behavior is expected for a ultrarelativistic beam; where the electromagnetic effect often plays an important role. In this letter, the influence of electromagnetic effects on the two stream instability is investigated for a relativistic electron beam propagating through a collisionless plasma channel.

The analysis in this paper is carried out within the framework of a macroscopic cold fluid model assuming that the beam-plasma fluids are immersed in a uniform axial magnetic field. For purposes of analytic simplification, the stability analysis is specialized to the case of a sharp-boundary equilibrium in which the beam and plasma channel have rectangular density profiles, i.e.,

$$n_j^0(r) = \begin{cases} \hat{n}_j = const., & 0 < r < R_b, \\ 0, & R_b < r \, R_c, \end{cases} \qquad (1)$$

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where the subscript $j = b, i, e$ denote beam electrons, plasma ions and electrons, respectively, $r = R_c$ is the radial location of a grounded conducting wall. Since the unstable mechanism of the two stream instability is mostly due to the fluctuations of the axial electric field, the stability analysis in this article is restricted to perturbations of the transverse magnetic mode polarization. In addition, it is further assumed that the perturbations are axisymmetric ($\partial/\partial\theta = 0$). This is a reasonable assumption because the axisymmetric perturbation is the most unstable mode for this kind of instability. Combining all of these assumptions and restrictions, it is straightforward to show that the eigenvalue equation of the two stream instability is expressed as

$$\left(\frac{1}{r}\frac{d}{dr} r \frac{d}{dr} + \frac{\omega^2}{c^2} - k^2\right) \delta\hat{E}_z(r) = 4\pi i k \left[\delta\hat{\rho}(r) - \frac{\omega}{c^2 k} \delta\hat{J}_z(r)\right], \qquad (2)$$

where $\delta\hat{E}_z(r)$ is the perturbed axial electric field, $\delta\hat{\rho}(r) = \sum_j e_j \delta\hat{n}_j$ and $\delta\hat{J}_z(r) = \sum_j e_j(v^0_{jz}\delta\hat{n}_j + n^0_j\delta\hat{V}_{jz})$ are the perturbed charge and the axial component of the current densities, respectively, e_j is the charge, n^0_j is the equilibrium density, v^0_{jz} is the mean equilibrium axial velocity, $\delta\hat{n}_j$ is the perturbed density and $\delta\hat{V}_{jz}$ is the perturbed axial velocity, of species j, ω is the complex eigenfrequency and k is the axial wavenumber.

Consistant with polarizations of the two stream instability, the transverse component of the perturbed electromagnetic field is neglected on the present analysis. This approximation tremendously simplifies the linearized fluid calculation of the equation of motion and continuity equation for the beam and plasma fluid element. Thus, after a straightforward calculation, the perturbed axial velocity is expressed as

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$$\delta\hat{V}_{jz} = i \frac{e_j}{\gamma_j^3 m_j} \frac{1}{\omega - kV_{jz}^0} \delta\hat{E}_z,$$ (3)

where m_j is the rest mass of particles of species j. Similarly, the perturbed density is given by

$$\delta\hat{n}_j = k \frac{n_j^0}{\omega - kV_{jz}^0} \delta\hat{V}_{jz}.$$ (4)

More detail calculation including the transverse component of the electromagnetic fields are currently under investigation by the author and will be presented elsewhere. Note from Eqs. (3) and (4) that both the perturbed density $\delta\hat{n}_j$ and axial velocity $\delta\hat{V}_{jz}$ profiles can eventually be expressed in terms of the perturbed axial electric field $\delta\hat{E}_z$. Substituting Eqs. (3) and (4) into Eq. (2) and carrying out a straightforward calculation, the eigenvalue equation in Eq. (2) is expressed as

$$\left\{\frac{1}{r}\frac{d}{dr} r \frac{d}{dr} - \left(k^2 - \frac{\omega^2}{c^2}\right)\left[1 - \sum_j \frac{\omega_{pj}^2(r)/\gamma_j^3}{(\omega - kV_{jz}^0)^2}\right]\right\} \delta\hat{E}_z(r) = 0,$$ (5)

where $\omega_{pj}^2(r) = 4\pi n_j^0(r)e_j^2/m_j$ is the nonrelativistic plasma frequency-squared of particles of species j. Note that $\omega_{pj}^2(r) = \hat{\omega}_{pj}^2 = $ const., for $0 < r < R_b$ and zero, otherwise.

For the present purposes, the eigenvalue equation (5) is solved for long wavelength perturbations characterized by

$$\left|k^2 - \frac{\omega^2}{c^2}\right|R_c^2 << \left|k^2 - \frac{\omega^2}{c^2}\right|R_b^2\left|1 - \sum_j \frac{\hat{\omega}_{pj}^2/\gamma_j^3}{(\omega - kV_{jz}^0)^2}\right| < 1.$$ (6)

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Within the context of Eq. (6), the physically acceptable solution to Eq. (5) can be approximately given by

$$\delta\hat{E}_z(r) = \alpha \begin{cases} 1, & 0 < r < R_b, \\ -\ell n(r/R_c)/\ell n(R_c/R_b), & R_b < r < R_c, \end{cases} \tag{7}$$

where α is a constant and $R_c >> R_b$ is assumed in order to satisfy Eq. (6) self-consistently. Multiplying Eq. (5) by $r\delta\hat{E}_z(r)$ and integrating over r from $r = 0$ to $r = R_c$, it is straightforward to obtain the dispersion relation

$$\left[\frac{\hat{\omega}_p^2/\gamma_b^3}{(\omega - k\beta_b c)^2} + \frac{f_e\hat{\omega}_p^2}{\omega^2}\right] \left(k^2 - \frac{\omega^2}{c^2}\right) = \left(k^2 - \frac{\omega^2}{c^2}\right) + \frac{2}{R_b^2\ell n(R_c/R_b)}, \tag{8}$$

where use has been made of Eq. (7). In Eq. (8), $\hat{\omega}_p^2 = 4\pi e^2\hat{n}_b/m$ is the nonrelativistic plasma frequency-squared for beam electrons, $f_e = \hat{n}_e/\hat{n}_b$, and the ion contributions are neglected. In obtaining Eq. (8), it is assumed that the beam electrons have the same axial velocity $\beta_b c$ which is related to the mass ratio γ_b by $\gamma_b^2 = (1 - \beta_b^2)^{-1}$. Axial velocity of the plasma electrons is assumed to be zero. Within the context of Eq. (6), the term $k^2 - \omega^2/c^2$ in the right-hand side of Eq. (8) can be neglected. However, I keep this term for the clarity in the subsequent stability discussion, without affecting the final results in this letter.

Two points are noteworthy from Eq. (8). First, it is obvious that the term $2/R_b^2\ell n(R_c/R_b)$ in Eq. (8) represents the influence of finite radial geometry. Somehow, neglecting this term recovers the familiar one dimensional dispersion relation for the two stream instability. Second, the influence of electromagnetic effects are incorporated into the terms proportional to ω^2/c^2 in Eq. (8). Neglecting these terms gives the dispersion relation

$$\left[\frac{\hat{\omega}_p^2/\gamma_b^3}{(\omega - k\beta_b c)^2} + \frac{f_e\hat{\omega}_p^2}{\omega^2} - 1\right] k^2 = \frac{2}{R_b^2 \ln(R_c/R_b)} \tag{9}$$

for the electrostatic approximation. Paralleling the standard method developed in the previous literature,[2,3] the critical beam current for instability is analytically found from Eq. (9) and is given by

$$I_{ce} = \frac{I_A}{2\ln(R_c/R_b)} \frac{\beta_b^3/f_e}{\left(1 + 1/\gamma_b f_e^{1/3}\right)^3}, \tag{10}$$

for the electrostatic approximation, where $I_A = mc^3/e \approx 17000$ ampere is the Alfven critical current. The electron beam with current below the critical current is stable whereas the beam with current above the critical current is unstable. Note from Eq. (10) that the critical current is inversely proportional to the plasma electron density(f_e).

In order to demonstrate the influence of electromagnetic effects on stability behavior, Eq. (8) is numerically solved for complex eigenfrequency $\omega = \omega_r + i\omega_i$, determining the stability boundaries in $(\hat{\omega}_p, \gamma_b)$ parameter space for specified values of f_e. The critical current I_{cm} of the electromagnetic calculation is obtained from these stability boundary curves. For convenience in the subsequent analysis, we define the electromagnetic current enhancement ξ by

$$\xi = I_{cm}/I_{ce} \tag{11}$$

which is a function of γ_b and f_e in general. Shown in Fig. 1 is the electromagnetic current enhancement ξ versus the beam energy γ_b obtained from

Eqs. (8) and (11) for $R_c/R_b = 7.4$, and $f_e = 1$ and $f_e = 10$. Obviously from Fig. 1, the critical current for the electrostatic approximation is valid only for a mildly relativistic electron beam. The electromagnetic current enhancement ξ increases drastically with increasing values of energy γ_b and plasma density f_e, thereby exhibiting a substantial enhancement of the critical current by the electromagnetic effects for a relativistic beam.

It is instructive to rewrite Eq. (8) by

$$f(\omega) = \frac{\hat{\omega}_p^2/\gamma_b^3}{\left(\omega - k\beta_b c\right)^2} + \frac{f_e\hat{\omega}_p^2}{\omega^2} + \frac{2c^2/R_b^2 \ell n(R_c/R_b)}{\omega^2 - k^2 c^2} = 1, \tag{12}$$

which is plotted in Fig. 2. When the value of the local minimum point labeled P is less than unity, Eq. (12) has six real roots and the system is stable. It is noted from Eq. (12) that when β_b approaches to unity, the strength of the destabilizing term proportional to $\hat{\omega}_p^2/\gamma_b^3$ reduces drastically in comparison with the stabilizing term proportional to $2c^2/R_b^2 \ell n(R_c/R_b)$. In Fig. 2, the influence of vertical line $\omega = kc$ drastically pull down the local minimum point P as the line $\omega = k\beta_b c$ approaches to $\omega = kc$, eventually providing strong stabilizing effects. The stabilizing influence of electromagnetic effects originates from the perturbed axial current density $\delta\hat{J}_b(r)$ in Eq. (2).

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REFERENCES

1. Krall, N. A., and Trivelpiece, A. W., *Principles of Plasma Physics*, (New York: McGraw-Hill, 1973), Chapter 9.

2. Davidson, R. C., *Theory of Nonneutral Plasmas*, (Mass.: Benjamin Reading, 1974), Chapter 2.

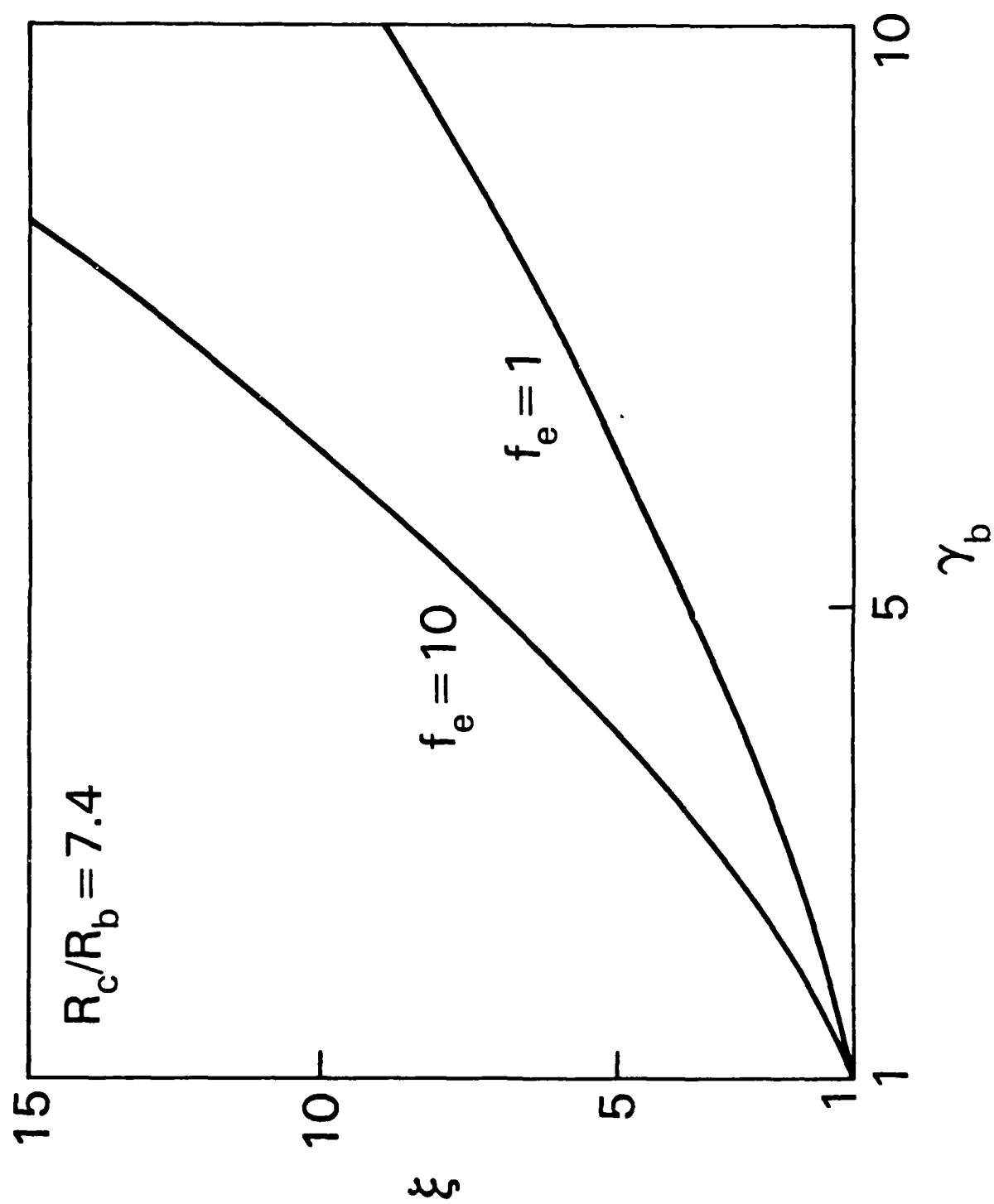3. Bogdankevich, B. L., and Rukhadze, A. A., *Soviet Physics USPEKHI*, 14, 163, 1971.

Figure 1

Figure 2

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